ENERGY AND MOMENTUM FLUXES THROUGH THE SEA SURFACE

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ABSTRACT

We use recent progress in our understanding of the energy and momentum balance in wind driven surface waves, to give a new estimate for the energy and momentum flux to and from the waves.

INTRODUCTION

It is generally assumed that to a given wind speed and stratification, there corresponds a definite turbulent transport of momentum and mechanical energy downward through the lowest part of the atmospheric boundary layer over sea. A large fraction of these fluxes (nearly 100% in fully rough flow) is used to grow the surface waves. However, this energy can not be retained by the waves, and a sizable fraction is passed on to the underlying ocean, quasi-instantaneously. The processes by which this dissipation takes place are insufficiently understood, but whitecapping seems to be a dominant mechanism.

The momentum flux in particular, tends to be dominated by what is happening at high frequencies, i.e. with the short waves. Therefore, in the past, people have tried to calculate these fluxes on the basis of empirical estimates of spectral levels at high frequency (see e.g. Huang, 1986) or by other means (Mitsuyasu, 1985 and Donelan, 1979). Recently, our understanding of the energy and momentum balance in the energy containing range of the spectrum has increased greatly (Komen et al, 1984; Hasselmann et al, 1987). Therefore, we are now in a better position to give flux estimates over the full spectral range.

After a short review of the recent progress we will estimate fluxes throughout the wave spectrum: at high frequencies from a simple "tail"-parametrization, but at lower frequencies from the physical balance, between sources and sinks in the energy and momentum budget of the surface waves.

Denoting momentum fluxes by τ and energy fluxes by Φ, we can formally write

$$\tau_a = \tau_{aw} + \tau_{ao} , \quad \tau_o = \tau_{wo} + \tau_{ao} \tag{1a}$$

$$\Phi_a = \Phi_{aw} + \Phi_{ao} , \quad \Phi_o = \Phi_{wo} + \Phi_{ao} \tag{1b}$$

Here τ_a is the momentum flux downward through the atmospheric boundary layer; τ_{aw} is the momentum flux going into waves, whereas τ_{ao} is the difference, creating such things as the drift current at the surface; τ_o is the total momentum flux into the ocean, τ_{wo} from the waves and τ_{ao} directly from the atmosphere. The same conventions apply to Φ. One should note that there is an important difference between energy and momentum fluxes. The momentum flux in the lower part of the atmospheric boundary layer is thought of as being constant over some height range, forces being absent. The energy flux is affected by viscous dissipation at every height. In this note we will concentrate on τ_a, τ_{aw}, τ_{wo}, Φ_{aw}, Φ_{wo}. As mentioned above we believe that

$$\tau_{ao} << \tau_a \ , \quad \Phi_{ao} << \Phi_a \tag{2}$$

in realistic situations, but we have not actually checked this in any way. More detailed investigations of the physical microscale aspects have recently been made by Chalikov (1986) and Hsu et al. (1982).

THE ENERGY BALANCE IN OCEAN WAVES

There is strong interest in ocean wave prediction, and the equation describing the evolution of ocean waves has been known for over 25 years now (Hasselmann, 1960). Yet, only recently have wave researchers been able to actually compute the wave spectrum from first principles, starting from this equation.

We denote the variance wavenumber spectrum by $F(\underline{k}\ ;\ \underline{x}\ ,\ t)$ with $\underline{k}$ the wave vector, and $\underline{x}$ and t place and time. It is normalized as

$$\int_{-\infty}^{+\infty} F(\underline{k})\, d\underline{k} = \int_0^\infty k dk \int_0^{2\pi} d\theta\, F(k,\theta) = \langle\eta^2\rangle \tag{3}$$

with η the surface elevation. It satisfies the following equation

$$\frac{\partial F}{\partial t} + \underline{\nabla} c_g F = S = S_{in} + S_{nl} + S_{ds} \tag{4}$$

This says that when you consider the spectral level for a given frequency and direction, it does not change if you move with the appropriate group velocity $\underline{c}_g$ except for the effect of the source terms S. They represent energy gain due to wind (S_{in}), weakly nonlinear resonant interactions among different wave components (S_{nl}) and energy loss due to dissipation (S_{ds}).

S_{in} has been estimated theoretically by several authors. The idea is that turbulent shear flow with a free internal boundary is unstable. The first successful calculation was done by Miles (1957). Later his methods have been refined. In addition, growth rates have been carefully measured (Snyder et al, 1981).

The weakly nonlinear resonant interactions have been calculated from first principles by Hasselmann (1961). This same author has also given an expression for the dissipation source term S_{ds}, based on an estimate of the whitecapping contribution (Hasselmann, 1974).

Even with all of these tools available, it was not so easy to make a wave prediction model based on the integration of (4). In fact, all of the models participating in the so-called Sea Wave Modelling Project (SWAMP, 1985) had to make some additional simplifying assumptions about the spectral shape and/or the evolution of spectral parameters. This led to contradictory results, especially in complex non-stationary situations with rapidly turning winds and wind-sea/swell transitions.

To overcome these problems an international group of wave researchers decided to jointly develop a wave model based on direct integration of eq. (4). This group, the WAM (= Wave Modelling) group has made considerable progress. A first version of the model has been run successfully on the CRAY-XMP/48 of the European Centre for Medium Range Weather Forecasts in Reading, U.K. It has been installed in both a regional and a global version, and it has been applied in both hind- and forecasting modes. (Komen, 1986). A full account of the model is in preparation (Hasselmann et al, 1987).

For the present application it is important to give the source terms of the WAM model. The wind input is taken as

$$S_{in} = \gamma_{in} F$$

$$\frac{\gamma_{in}}{\omega} = 0.25\ \varepsilon \max\left(28 \frac{u_*}{c} \cos\phi - 1,\ 0\right) \qquad (5)$$

Here $\omega = 2\pi = (gk)^{1/2}$ for gravity waves, $g = 9.8\ m/s^2$ being gravitational acceleration, ε is the ratio of the density of air and seawater, u_* is the friction velocity in the air, c is the phase velocity ($c = \omega/k$)) and ϕ is the angle between wind and waves. The nonlinear transfer, which conserves overall energy and momentum, but moves it from one wavenumber to another, is calculated numerically in the so-called discrete interaction approximation, (Hasselmann et al, 1985). The dissipation, finally, is taken as

$$S_{ds} = -\gamma_{ds} F$$

$$\frac{\gamma_{ds}}{\omega} = 1.59\ (\omega/\bar{\omega})\ \hat{\alpha}^2 \qquad (6)$$

$$\hat{\alpha} = \frac{\langle\eta^2\rangle \bar{\omega}^{-4}}{g^2}$$

which is quasi-linear in the spectrum, $\langle\eta^2\rangle$ and the mean angular frequency $\bar{\omega}$ expressing global properties of the spectrum.

FLUXES VERSUS SOURCE TERMS

Ocean waves carry energy and momentum, which can be expressed in terms of the variance spectrum

$$E = \rho_w \int_{-\infty}^{+\infty} \frac{\omega^2}{k} F(\underline{k})\, d\underline{k} \qquad (7a)$$

$$\underline{P} = \rho_w \int_{-\infty}^{+\infty} \frac{\omega \underline{k}}{k} F(\underline{k})\, d\underline{k} \qquad (7b)$$

Here E is energy, $\underline{P}$ momentum and ρ_w the density of water. For ω we now read the more general expression $\omega = (gk + Tk^3)^{\frac{1}{2}}$, with T surface tension, which is also valid at very high frequencies when capillary effects become important. For the longer waves ($k^2 << g/T$) this reduces to the deep water gravity wave dispersion relation, for which the factor $\rho\omega^2/k$ reduces to the familiar ρg, translating variance spectrum into energy spectrum. An easy way of memorizing (7) is by observing that the energy spectrum can be written as ωA, with A the action density, and the momentum spectrum similarly as $\underline{k}A$.

Anyway, from (4) and (7) it follows that the fluxes from the atmosphere to the waves and from the waves to the ocean can be written as

$$\underline{\tau}_{aw} = \rho_w \int d\underline{k} \; \frac{\omega \underline{k}}{k} S_{in} \qquad (8a)$$

$$\underline{\tau}_{wo} = -\rho_w \int d\underline{k} \; \frac{\omega \underline{k}}{k} S_{ds} \qquad (8b)$$

$$\Phi_{aw} = \rho_w \int d\underline{k} \; \frac{\omega^2}{k} S_{in} \qquad (8c)$$

$$\Phi_{wo} = -\rho_w \int d\underline{k} \; \frac{\omega^2}{k} S_{ds} \qquad (8d)$$

It is important to realize that the integrals extend over all wave numbers. Therefore, we will now first investigate the high wavenumber behaviour of the integrands.

THE SHORT WAVE CONTRIBUTION

Crucial for an estimate of the convergence of the integrals (8) is the short wave behaviour of the spectral density. There has been a lot of discussion as to whether this was f^{-4} or f^{-5}. Before joining this discussion one should always clearly state what frequency range one is referring too. Originally, Phillips (1958) considered wave spectral levels between say 3 times the peak frequency and 1 Hz. The JONSWAP tail-fit was made between 1.3 and 2 times the peak frequency. In his early paper Phillips proposed an f^{-5} tail, on the presumption that the limiting spectral level is determined by hydrodynamic processes alone. On dimensional grounds one then obtains for the frequency-directional variance spectrum:

$$G(f,\theta) = \frac{\alpha_P\, g^2}{(2\pi)^4 f^5} I(\theta) \qquad (9)$$

with the directional distribution $I(\theta)$ still to be specified. In k-space this corresponds with a wavenumber spectrum ($Gdf = F\, kdk$)

$$F(k,\theta) = \frac{\alpha_P}{2k^4} I(\theta) \tag{10}$$

Later Toba (1973) suggested that the saturation level was not independent from the friction velocity and that in fact

$$G(f,\theta) = \frac{\alpha_T g u_*}{(2\pi)^4 f^4} I(\theta) \tag{11}$$

should be more appropriate. This idea was endorsed by Phillips (1985). Experimental data favour both. At not too high frequency Birch and Ewing (1986) observed f^{-4}; at higher frequencies they found f^{-5}. A crude estimate of the transition frequency/wavenumber was given by Peter Janssen (private communication), who obtained

$$k_{tr} = \frac{g}{u_*^2} \left(\frac{\alpha_P}{\alpha_T}\right)^2 \tag{12}$$

With typical values for the α's ($\alpha_P = 0.01$, $\alpha_T = 0.1$) the corresponding transition frequency is within the dynamic range of the WAM model. Therefore, in the following, we will consider WAM model spectra up to the transition frequency. Above, we will assume the f^{-5} tail.

We will now proceed by estimating the high frequency contribution to the momentum-flux from the atmosphere to the waves. Before doing this, however, we should point out that the wind input source term, eq. 5, is not applicable in the high frequency range. In fact, Plant (1982) presented a compilation of wave growth measurements, which turned out to be described by a quadratic u_*/c dependence,

$$\frac{\gamma_{in}}{\omega} = A \left(\frac{u_*}{c}\right)^2 \cos\phi \quad , \quad A = 0.04 \pm 0.02 \tag{13}$$

This fit is valid for growing waves and extends towards the highest frequencies. At lower frequencies it has to be merged with (5). So we split τ_{aw} in a high and a low frequency contribution

$$\tau_{aw} = \tau_{aw}^{l} + \tau_{aw}^{h} \tag{14}$$

τ_{aw}^{l} will be considered below. Here we compute τ_{aw}^{h} as

$$\underline{\tau}_{aw}^{h} = \rho_w \int_{k_{tr}}^{\infty} \frac{\omega \underline{k}}{k} A\omega\left(\frac{u_*}{c}\right)^2 \cos\phi \times \frac{\alpha_P}{2k^4} I(\theta) k dk d\theta \tag{15}$$

Taking the X-axis in the wind direction one finds $\underline{\tau} = (\tau, o)$ with

$$\tau^h_{aw} = \tfrac{1}{2} A \rho_w \alpha_p u_*^2 \int_{k_{tr}}^{\infty} \frac{dk}{k} \int d\theta \, I(\theta) \cos^2\theta \qquad (16)$$

The k-integral is divergent. Fortunately, the theory considered so far is incomplete at very high frequencies. There, viscosity damps the waves. Therefore, we introduce as an ultra-violet cutoff the wavenumber at which viscosity begins to dominate. This is at

$$\omega_v = \frac{A u_*^2}{4 \nu_w} \qquad (17)$$

with ν_w the viscosity of water. Not much is known about the angular distribution. To be specific we will assume a $\cos^2\theta$ distribution ($I(\theta) = (2/\pi) \cos^2\theta$). This then yields

$$\tau^h_{aw} = \frac{3}{8} \rho_w \alpha_p A u_*^2 \ln (k_v / k_{tr}) \qquad (18)$$

The pseudo-divergence, resulting in a $\ln k_v$ factor stresses the importance of the short wave contribution.

The energy can be dealt within a similar way. Performing similar manipulations one obtains

$$\Phi^h_{aw} = \frac{4}{3\pi} \rho_w \alpha_p A u_*^2 \int_{k_{tr}}^{k_v} \frac{\omega}{k^2} \, dk \qquad (19)$$

$$\omega = (gk + Tk^3)^{\frac{1}{2}}$$

This integral would also diverge for $k_v = \infty$, but here the divergence only appears in the ultra capillary limit. In the gravity range the integrand behaves as $k^{-3/2}$. Eq. (19) was evaluated by splitting the integration range in 3 parts: $(k_{tr}, k_o/3)$, $(k_o/3 , 3 k_o)$ and $(3 k_o , k_v)$ where $k_o = (g/T)^{\frac{1}{2}}$ is the wavenumber at which gravity and surface tension effects are equally important. In the first interval the waves are approximated as pure gravity, in the middle range the integral was computed numerically, and in the last interval the capillary limit was taken. Using $k_{tr} < k_o$ I obtained

$$\Phi^h_{aw} = \frac{4}{3\pi} \rho_w \alpha_p A u_*^2 \left\{ \frac{2g}{\omega_{tr}} + 2.41 \, (\omega_o / k_o) + 2 \, T^{\frac{1}{2}} \left(k_v^{\frac{1}{2}} - (3 k_o)^{\frac{1}{2}} \right) \right\} \qquad (20)$$

Because of the smallness of T ($7.2 \; 10^{-5} \; m^3 \, s^{-2}$) the last term is rather small. We will illustrate this by considering a friction velocity of 0.85 m/s. This value roughly corresponds to a windspeed of 20 m/s. It will be used as a reference in all numerical estimates to follow. We find

$$k_{tr} = 0.14 \quad , \quad k_o = 369 \quad \text{and} \quad k_v = 8360 \quad m^{-1}$$

and

$$\tau^h_{aw} = 1.19 \text{ Pa} \tag{21a}$$

$$\Phi^h_{aw} = 0.12\ (16.7 + 0.56 + 0.98)$$

$$= 2.2 \text{ W/m}^2 \tag{21b}$$

It is interesting to note that τ^h_{aw} exceeds $\tau_a = \rho_a u_*^2$ by 40%. We will discuss this result below, after the estimate of the low frequency contribution.
So far we have only considered fluxes from the atmosphere to the waves. To estimate the fluxes from the waves to the ocean we would have to use (8b) and (8d) for high frequencies. Unfortunately, S_{ds} is not known there. Therefore, we will use the fact that at high frequencies there is a balance between the source terms

$$- \int_{k_{tr}}^{k_v} \frac{\omega^2}{k} S_{ds}\, d\underline{k} = \int_{k_{tr}}^{k_v} \frac{\omega^2}{k} (S_{in} + S_{nl})\, d\underline{k} =$$

$$= \int_{k_{tr}}^{k_v} \frac{\omega^2}{k} S_{in} d\underline{k} - \int_{o}^{k_{tr}} \frac{\omega^2}{k} S_{nl}\, d\underline{k} \tag{22}$$

The second line follows because the nonlinear transfer conserves energy. A similar expression holds for the momentum flux. Below we will give a numerical estimate of (22).

LONG WAVE CONTRIBUTION

We have calculated long wave contributions by using results of the WAM model. This is a relatively straightforward calculation, because, once you run the model, you have spectra and source terms every time step in every grid point. It should be emphasized that in general the fluxes depend on local wind speed as well as on details of the spectral shape, which are determined by the geometry of the basin and by the time history of the windfield. A full analysis was outside the scope of this note. We have only analyzed fetch limited growth, in which a constant windfield blows off-shore, perpendicular to a straight coast, and in which the stationary response is considered. The results are given in table 1 for two different fetches, one in which the waves are still actively growing (X = 50 km, H_s = 4 m) and one in which saturation is being approached (X = 500 km, H_s = 7.5 m)

Table 1 Modelled fluxes of energy Φ and momentum τ into (aw) and from (wo) the energy containing waves $[k<k_{tr}$, eq. (12)$]$ at two different fetches. Φ_{nl} and τ_{nl} denote the nonlinear transfer from long waves to short waves as modelled with the discrete interaction approximation. ($u_* = 0.85$ m/s).

X(km)	Φ^l_{aw}	Φ^l_{wo}	Φ_{nl}	τ^l_{aw}	τ^l_{wo}	τ_{nl}
	(W/m^2)				(Pa)	
50	1.5	0.6	0.1	0.13	0.04	0.02
500	2.4	1.7	0.7	0.19	0.10	0.09

At the shorter fetch the energy and momentum excess makes the waves grow. In the steady state the excess is advected away, making the waves higher at longer fetch. At the longer fetch there is a balance between energy gain and energy loss.

DISCUSSION

First of all let us combine the results of the previous two sections. We then obtain the following picture

Table 2. Modelled energy fluxes (W/m^2) to (aw) and from (wo) the waves, l denotes long waves, h indicates the short wave contribution ($u_* = 0.85$ m/s).

X(km)	Φ^l_{aw}	Φ^h_{aw}	Φ_{aw}	Φ^l_{wo}	Φ^h_{wo}	Φ_{wo}
50	1.5	2.2	3.7	0.6	2.3	2.9
500	2.4	2.2	4.6	1.7	2.9	4.6

Table 3. Modelled momentum fluxes (Pa). The meaning of sub and superscripts is as in table 2.

X(km)	τ^l_{aw}	τ^h_{aw}	τ_{aw}	τ^l_{wo}	τ^h_{wo}	τ_{wo}
50	0.13	1.19	1.32	0.04	1.21	1.25
500	0.19	1.19	1.38	0.10	1.28	1.38

In the tables Φ^h_{wo} and τ^h_{wo} have been calculated using (22).

A few remarks can be made:

(i) Both energy and momentum flux to and from the waves increase with fetch.

This is because the level in the tail was fixed, and for larger fetch more wave components take part in the transfer. The dissipation increases more strongly then the input, a necessary condition for reaching equilibrium.

(ii) The momentum fluxes are dominated by high frequency contributions, much more than the energy fluxes.

(iii) There is something "wrong" with the magnitude of τ_{aw}. For example at X = 500 km we have

$$\tau_{aw} = 1.7 \times \tau_a$$

implying that the waves would receive more momentum then the boundary layer provides. We distinguish two possibilities

a. Our estimate is correct. This is possible when a deceleration extracts momentum from the lowest part of the atmospheric boundary layer. Although τ_a is defined as the momentum flux at the lower boundary of the boundary layer, it will be measured in practice at, say, 10 m height. Over land the flux is then constant down to the surface. Over waves this might be different. In fact it could be that over sea the flux to short waves is suppressed as has been suggested by Janssen (1982) (see also Chalikov, 1986).

b. Our estimate is wrong. One should note that the high frequency contribution alone already exceeds τ_a, so this is suspect. One should realize that our knowledge of source terms and spectra at high frequencies in the field is still fragmentary. Plant's constant is A = 0.04 ± 0.02, which implies a 50% error in our estimate of τ_a. Also the spectral level was taken as in (10) with α_p = 0.01, which also is only true to a certain accuracy. One of the weakest assumptions was the $\cos^2\theta$ angular distribution at high frequency. There are indications, also in the WAM model, that this might be considerably flatter. (An explanation of this is perhaps found in the behavior of the nonlinear transfer at high frequencies). This could reduce our estimate by a factor of 2 or 3.

CONCLUSION

Although our understanding of the energy balance in wind driven ocean waves has improved greatly, our ability for estimating energy and momentum fluxes to and from the waves is still limited. For the energy flux from waves generated by a wind of about 20 m/s (u_* = 0.85 m/s) we typically find a few W/m^2. This is large compared to the fluxes currently studied inside the mixed layer, so one wonders what happens to this energy. Part will go into mean motion, an other part will dissipate, but the rest will certainly penetrate as turbulence. How this exactly happens is a challenge for mixed layer modelers.

The momentum flux through the energy containing waves is relatively small. When we estimated the total flux we made a number of assumptions about source terms and spectral shapes at high frequencies. These require further confirmation. Our answer came out slightly larger than what would have been considered acceptable. The discrepancy can be easily explained for instance by a flat angular distribution; it could also hint at something we don't understand about the lowest meter or so of the atmospheric boundary layer over sea.

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