# Can one discriminate between different wave generation mechanisms on the basis of wind profile measurements?


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## Abstract

In preparation for field experiments with a wave follower we have compared predictions from three different wave generation theories: the quasi-laminar theory of Miles and two turbulent theories. In the first approach the evolution is governed by the Rayleigh equation. In the other approaches the Reynolds equations need to be solved, which involves a closure assumption. We have considered two cases, namely a mixing length (ML) closure scheme and a Reynolds stress model. The calculations are made for a monochromatic wave. Our calculations are given as profiles of the wave-induced velocity and Reynolds stress perturbations, and we conclude that measurements of the (periodic) velocity perturbations can discriminate between different theories. We also compute the (mean) wave-induced momentum flux, which, for a monochromatic wave, discriminates well between the different models. However, this flux is hard to observe in the field because the stress is also supported by the shorter background waves which are always present during active wave generation. When we take this into account we find that the total wave stress becomes important only at heights that are too low to be measured with the present instrument. Therefore, measurements should focus on the wave-induced velocity and Reynolds stress perturbations.

# 1 Introduction

In the last decade there has been considerable progress in our understanding of the generation of wind waves, and in particular in our understanding of the interaction between turbulence and wave-induced air motion. Numerical simulations of wave-induced velocity and Reynolds stress perturbations have been compared with laboratory observations (cf., for example, Stewart, 1970 and Mastenbroek et al., 1996). The results are encouraging. A comparison in the field is still lacking.

At KNMI we are developing a wave follower, which will allow the measurement of velocity and stress profiles close to the sea surface, even below the level of the wave crests. In preparation for the first tests with this instrument, we have computed predictions of the wave-induced velocity and Reynolds stress perturbations from three different theories: the quasi-laminar theory of Miles (1957) and two turbulent models involving a closure assumption. We considered both a mixing length (ML) closure scheme and a scheme based on the Reynolds stress model of Launder, Reece and Rodi (LRR, 1975). This latter approach is consistent with rapid distortion ideas developed by Belcher and Hunt (see, for example, Belcher and Hunt, 1998, for a review.)

# 2 The wave follower

Our wave follower is being developed for use at research platform Meetpost Noordwijk, in the North Sea, 9 km off the Dutch coast. The instrument will have a total stroke of 3.5 m, which in principle should allow measurements to be made for wind speeds (at 10 m height, $U_{10}$) of up to 15 m/s. The instrument has been designed to follow waves with a wavelength of 1 m and longer with an accuracy of 10 cm. The deviation from the actual surface will be registered accurately. A set of pressure anemometers (PA; Oost et al, 1990) will be mounted as an integral part of the actual wave following element. This is done to reduce flow distortion effects, which can be very serious if sizable instruments are mounted close to the water surface. PA's measure all components of the wind speed with a 20 Hz sampling frequency and an accuracy of a few cm/s. The PA's will be mounted in such a way that their mutual distance is a multiple of 30 cm. Other instruments can be clamped to the wave following pole in arbitrary positions. The minimum distance of a sensor to the water surface is about 10 cm. This low distance requires mounting of a special wave height sensor below the wave follower and can only be used for wind speeds not exceeding 5 - 6 m/s. At higher wind speeds we will typically be measuring velocities in the height range from 0.5 m to 2 m.

# 3 The wave boundary layer

Air flow over a surface is described by

$$u_t + uu_x + wu_z = -p_x + \tau^{11}_x + \tau^{12}_z, \tag{1}$$

$$w_t + uw_x + ww_z = -p_z + \tau^{21}_x + \tau^{22}_z, \tag{2}$$

$$u_x + w_z = 0. \tag{3}$$

Here $(u, w) = u_i(i = 1, 2)$ denotes velocity, $p$ denotes pressure and $\tau^{ij} = -\overline{u'_i u'_j}$ is the Reynolds stress. We use the subscripts $x, z$ and $t$ to denote differentiation.

Now consider stationary boundary layer flow over a harmonic wave profile given by $\eta = a\cos[k(x - c\,t)]$ plus possibly Stokes corrections. In a lowest order perturbation approach eqns. 1 - 3 then reduce to

$$Wik\tilde{u} + W_z\tilde{w} = -ik\tilde{p} + ik\tilde{\tau}^{11} + \tilde{\tau}^{12}_z, \tag{4}$$

$$Wik\tilde{w} + W_z\tilde{w} = -\tilde{p}_z + ik\tilde{\tau}^{21} + \tilde{\tau}^{22}_z, \tag{5}$$

$$ik\tilde{u} + \tilde{w}_z = 0. \tag{6}$$

Here we transformed to a coordinate frame moving with the wave propagation speed, introduced the notation $W = U - c$, and defined velocity and Reynolds stress pertubations

$$\Delta u_i = \text{ Re } \{\tilde{u}_i e^{ikx}\}, \tag{7}$$

$$\Delta\tau^{ij} = \text{ Re } \{\tilde{\tau}^{ij} e^{ikx}\}. \tag{8}$$

An important quantity is the total vertical momentum flux which is the sum of a part supported by turbulent motion ($\tau_t = -\overline{u'w'}$ ) and a part supported by the wave induced motion ( $\tau_w = - < \Delta u \Delta w >$, in a Cartesian coordinate system, where the brackets denote the average value over the wave):

$$\tau = \tau_t + \tau_w \tag{9}$$

Miles, in his quasi-linear approach, neglected the wave-induced Reynolds stress perturbations. The dynamics is then described by a Rayleigh equation which follows immediately from eqns (4) - (6). We solved the Rayleigh equation numerically, however, matching to the analytical solution near the critical height $z_c$, defined by $W_0(z_c) = 0$ (See Janssen, 1982 and Komen et al, 1994).

In more recent approaches the Reynolds stress perturbations are retained. We have considered two cases. In the mixing length case, the Reynolds stress is expressed in terms of the local velocity shear. However, this is believed to be less appropriate away from the surface where advection of turbulence by the mean flow becomes important. Therefore, the second case takes advection into account through the following equation:

$$\frac{\partial \overline{u_i' u_j'}}{\partial t} + \bar{u}_k \frac{\partial \overline{u_i' u_j'}}{\partial x_k} = P_{ij} + T_{ij} + \Pi_{ij} - \epsilon_{ij}. \tag{10}$$

The different terms on the right side describe production and dissipation of turbulence. To calculate them we used the LRR turbulence closure model, and we solved eqns. (1)-(3), (10) numerically in a wave-following coordinate system (for details see Mastenbroek, 1996). A constant background roughness length $z_0 = 0.014 u_*^2/g$, with $u_*$ the friction velocity and $g$ the gravitational acceleration, was assumed.

# 4 Model results

We have performed computations for a large number of realistic conditions. The results are given in terms of the wave following coordinate $z'$ which measures the distance to the actual sea surface. (However the velocities are NOT transformed). We will present two cases here. In the first case (Fig. 1) the wind speed is taken to be 15 m/s and the wave is relatively slow: $U_{10}/c = 2$. In the second case (Fig. 2) the wind is 10 m/s and the wave is relatively fast: $U_{10}/c = 1.2$. Each figure gives the amplitude and the phase of the vertical and horizontal wave-induced velocity perturbations, defined by $\Delta w = |\ \Delta w\ | cos(k \cdot x - \phi_w)$, and similarly for $\Delta u$. The dotted line is the prediction by Miles' theory, the dashed line is the mixing length solution and the solid line is the result of the Reynolds stress model. There are several striking features. In both cases the vertical air velocity perturbation is

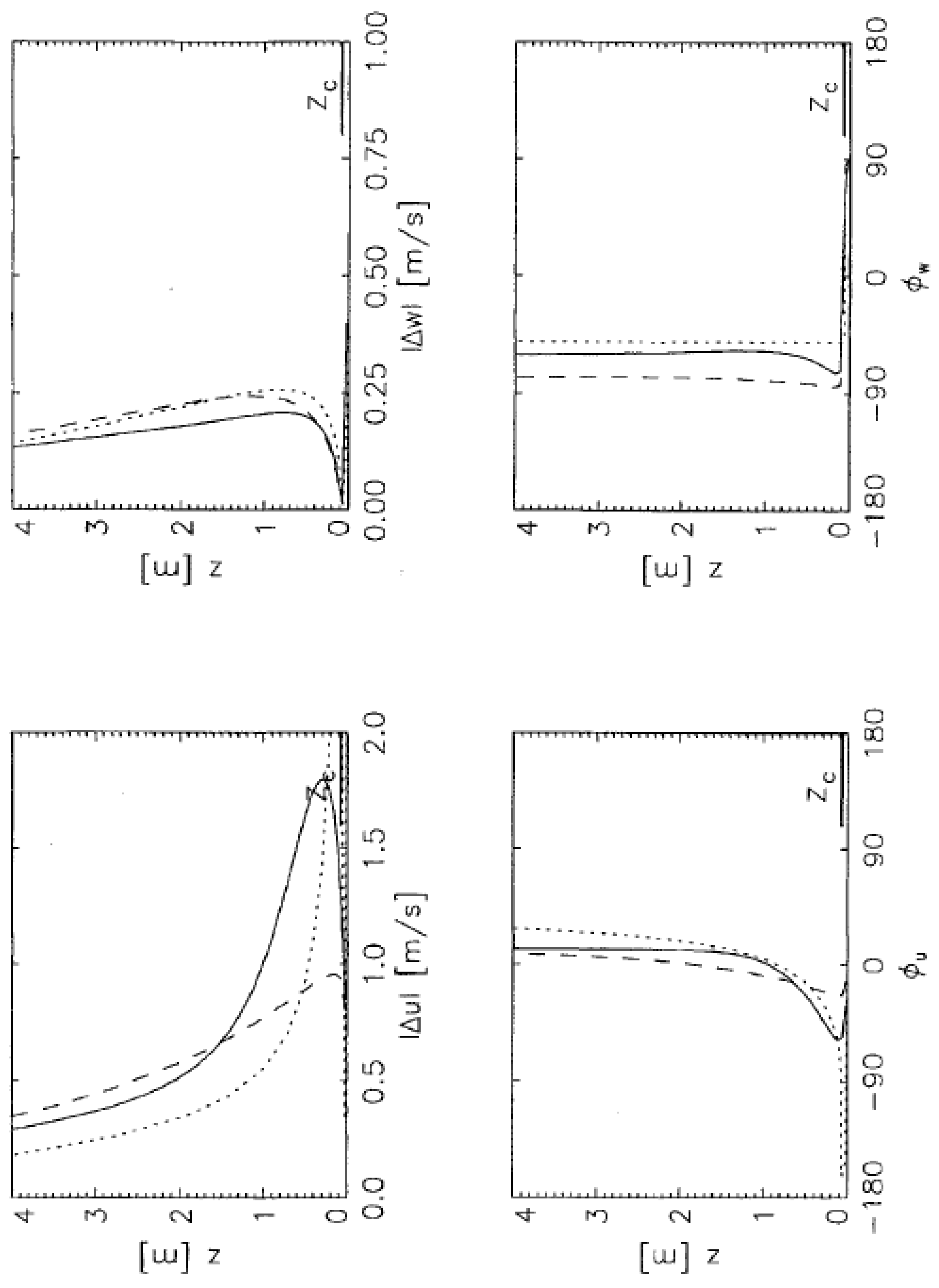


Figure 1: Wave-induced velocity perturbations for a "slow" wave ($U_{10}$ / c = 2). The wind speed was taken as 15 m/s. Given are the amplitude and the phase of the vertical and horizontal wave-induced velocity perturbations. The dotted line is the prediction by Miles'

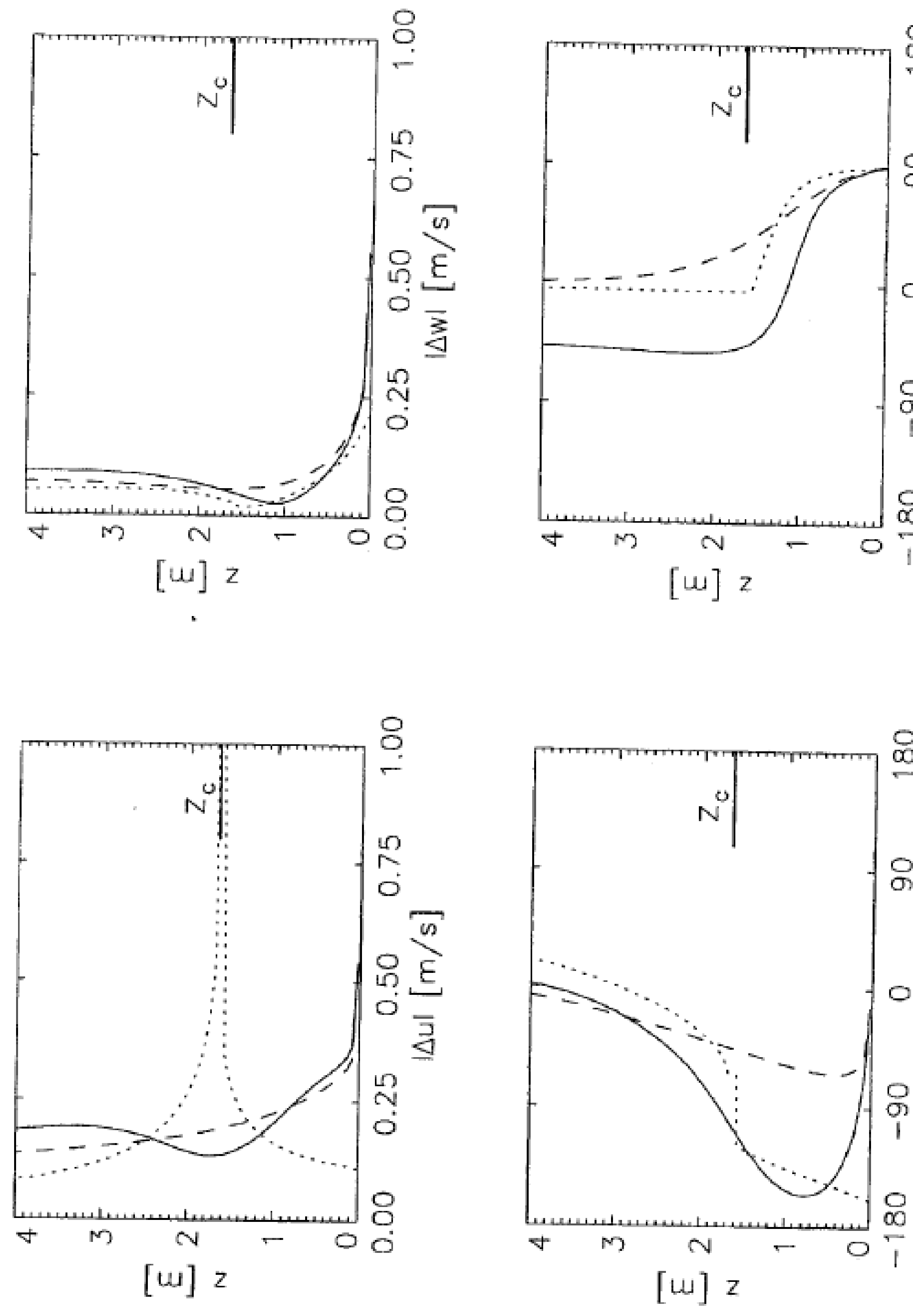


Figure 2: Wave-induced velocity perturbations for a "fast" wave ($U_{10}$ / c = 1.2). The wind speed was taken as 10 m/s. Conventions are as in **Fig.**1.

continuous at the water surface, which implies a phase of + 90 degrees. With increasing height this phase changes sign because the air flows down behind the crest. In the slow-wave case the magnitude decreases rapidly with height towards a minimum at the critical height (which is at only 7 cm). Higher up magnitude and phase are model dependent. A similar behaviour is seen in the fast-wave case. The Miles solution for the horizontal velocity perturbation becomes infinite at the critical height. This is of course artificial and due to the neglect of viscosity. It is interesting to note that our LRR solution predicts similar large horizontal velocity perturbations in the slow-wave case. In the fast-wave case the wave-induced horizontal velocity perturbations are similar in the ML and LRR case, but now the phase is remarkably different.

We have made a similar study of the wave-induced Reynolds stress perturbations and there we find similar differences and agreements.

We have also studied the effect of waves on the mean boundary layer properties. A typical parameter is $\tau_w/\tau$. We found that this quantity becomes observable only at a height of less then, typically, 10 cm, which is too low to be measured with the present instrument.

# 5 Conclusions

We described a wave follower, under construction at KNMI. From an analysis of predictions of three existing theories for the turbulent wave boundary layer we conclude that this instrument can be used to discriminate between the different predictions. We also found that it is sensible to concentrate on wave-induced velocity and stress perturbations, as we do not anticipate to see deviations from the wave-averaged structure of the boundary layer.

We used the theories only as guidance for the design of an experiment and did not discuss their validity. In fact, there is good reason to doubt this validity, because even the theory with the best theoretical foundation is known to lead to an underprediction of the wave growth parameter. Therefore, we believe there is room for theoretical improvement and we hope that our measurements will contribute to a better understanding of the turbulent wave boundary layer.

## Acknowledgements

We would like to thank Peter Janssen and Gerrit Burgers for making available their Rayleigh solver and Ed Worrell for providing the technical specifications for the wave follower.

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